



OPTIMIZATION OF THE INVENTORY BY USING SIMULATION

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Abstract: The article deals with the simulation, more specifically with the method Monte Carlo. The term as a simulation, simulate, a simulator are well known in many scientific disciplines. Simulation, especially computer simulation has been in a rapid growth in recent years. The simulation is experimenting with computer models based on the real production process in order to optimize the production processes or the system. The simulation model allows to perform a number of experiments, analyze them, evaluate, optimize and afterwards apply the results to the real system. By using the Monte Carlo method it is possible to find the optimal quantity of reserves and to maximaze a profit.

1 Introduction

The term simulation is used more often but usually as a synonymous of imitate or pretend. The object, which to some extent imitates the functionality, condition or processes a certain imitation object (as a driving simulator, a simulator of a free fall) refers to as a simulator. With the term of simulation, simulate, simulator we are coming across in many scientific disciplines. Moreover, it is used in various fields: computer systems, logistics and production systems, financial and econometric models, space travel, military operations, urban systems, etc.

2 Modeling and simulation

During designing of production processes many challenges occur, for instance how to solve problems in the process, how to innovate or improve the process. One possibility is to simulate the production process. The simulation is experimenting with computer models based on the real production process in order to optimize the production processes, the entire processes or a system. It enables to experience the behavior of the system after implementation and to see the possible future scenarios and uncover problems. Simulation is not a tool which provides an optimal solution to the problem but it is a support tool that tests the impact of our decisions on the simulated model. The simulation model allows you to perform a number of experiments, analyze them, evaluate, optimize and afterwards apply the results to the real system.

According to the definition a simulation is a research method, which comprised on the examination of a dynamic system. The dynamic system is replaced by a simulator, on which are afterwards performed experiments to obtain information about the original investigated system. It is a computer simulation, in which the simulator consists of a simulation program - model introduced to the computer.

By using the simulation it is possible to solve several problems, such as: the most convenient management system, the best lay-out of machines, the optimal size of the inventory, determine the number of machines and additional equipment, determining the optimal size of operating machines and equipment, determining the optimal dose size, determine the impact of failures and downtime the manufacturing process, and many other applications.

Some of the reasons for using of the simulation:

- > The need for predictions of future action,
- Restructuring of the manufacturing process,
- Confirmation of hypotheses,
- Reduce the risk of decision making,
- Increase the efficiency,
- Elimination of deficiency in the production process without interfering with the real processes,
- Impossibility of implementing the production process (production system does not exist yet, it is not possible to consider a greater number of variants, it is not possible to change any parameters),
- Cost-saving,
- Experiments can cause malfunctions in the equipment,
- Experimentation on a real object is expensive (in terms of money, time).



During the application simulations two basic rules are used:

> The benefits achieved by applying simulation should be greater than the costs required to implement simulation (quantitative and qualitative benefits). The simulation is justified if the direct benefits of simulation are more than the cost of the simulation. On the effectiveness of simulations has a large influence the time when the simulation is carried out.

> The simulation should be used in the initial stages of the project process, because the most potential improvements of the system can be achieved in the initial stages, while the costs involved are currently the lowest in the following phases. In the process of implementation and operation exists only a few degrees of freedom for a change. Proper and timely decisions are incomparably greater than the benefits of optimization in subsequent stages of the project.

Simulation is actually mathematical modeling; it means experiment on a real system is replaced by the solution of a mathematical model on a computer. The principle of computer simulations lies in the establishment of a system's simulation model and in the implementation of experiments with simulation model in the correct interpretation and application of the results achieved for improving the real system. The real system is a system which is the subject of our interest. The simulation model is a dynamic model (mathematical, logical, and so on.), which in a way shows current system; its structure and a set of rules under which the model in a way behave (generated data). The models can be divided into mathematical and physical. Before the simulation model is built, is necessary to define the goal of the simulation (cost reduction, reduction of production, optimize deployment of production facilities), and to obtain input data. For defining the goal is mainly used abstraction (by neglecting aspects and elements of the reduction characteristics, the links between them), structuring (decomposition of the sub-systems, while maintaining the hierarchy) and analogue (draw conclusions on the basis of similarities with other systems). The term simulation experiment is a set of simulation runs - multiple repetitions of the basic cycle simulation for a specific time period and parameter simulation with partial evaluation of results. It is therefore a set of the operation's simulation in which a change of input quantities are performed.

2.1 Simulation Monte Carlo

Monte Carlo is an algorithm for simulation systems. Monte Carlo method means the numerical solution of probabilistic and deterministic tasks using many times repeated probabilistic experiments. It is a stochastic method using pseudorandom numbers. The basic objective of is to determine the mean values resulting from the random phenomenon. In order to create a computer model of the phenomenon the sufficient amount of simulation data needs to be obtained by using statistical methods. There are two approaches to solve problems by the Monte Carlo approach based on geometric probability and an approach based on estimates of the mean random variables. The algorithm method is based on selecting a random number of the specified statistical distribution (Tab. 1) probability distribution if sufficient data can be assessed using the Pearson test (Chi-Square test), Kolmogorov-Smirnov test and a simple visual analysis histogram. In case when no available historical data exists, the probability distribution is determinate on professional judgment.

The actual simulation is a process of random selection of specific risk-factors of the predefined division. Simulation (number of simulation runs is recommended at least 1,000 times) selects a random number and applies it in the model definition. Randomness and objectivity is ensured by the random generator, thus pseudorandom numbers. The simulation output is a probability distribution of the target variable, so the solution is probabilistic in nature and is dependent on what the distribution of the individual values generated. After the simulation, the aggregate statistics of evaluation indicators are created - arithmetic mean, standard deviation, mode, median, and coefficient of variation, the likelihood of gains or losses. The results can be represented graphically.

Table 1 Statistical probability distribution:

Discrete Probability	Continuous Probability
Distributions	Distributions
Uniform distribution	Uniform distribution
Bernoulli	Normal Distribution
distribution	Standard normal
Binomial	distribution
distribution	➢ Exponential
Poisson distribution	distribution
> Hypergeometric	Log-normal
distribution	distribution
	➢ Weibull distribution
	Student distribution
	\succ χ^2 distribution
	Fisher-Snedecerove
	distribution
	➢ Gamma distribution

2.2 Simulation in Excel

To simulate models in different random processes can use widely available Microsoft Excel spreadsheet (insert function). For generating random numbers can use the command RAND (), we get a random number with uniform distribution in the interval (0,1). To determine the percentage of occurrence of random numbers, for example, in four intervals is used COUNTIF function. To generate the random number in the range, for example



from 500 to 1000, use the command INT (RAND) * 500 + 500. Press F9 for another simulation run. (Fig.1,2,3)



Figure 1 Generating random numbers



Figure 2 Determine the percentage of occurrence



Figure 3 A random numbers from the interval

The above examples can be applied for example to determine the optimal amount of the stock. The warehouse owner buys goods for 20 euros per package and sells it for 30 euros per pack, in the event that it was available on certain date remains merchandise and sells it with 50% discount. This means that if demand for goods greater than the quantity ordered he will come from a lack of profit, but if the demand for goods is less than the quantity ordered, he will lose profits due to discounts.

If the demand is (d) the product is higher than the ordered quantity (q) and the equation the result is

Z = (30 - 20)q.	(1)	
If the demand is (d) the product is less	than	the
quantity ordered (q) and the equation the result is	5	
Z=30 d-20q+15 (q-d).	(2)	

Demand is a probabilistic variable, as it is known, that cannot be determined profit. Suppose that the demand will be a number from $d \in (500, 1000)$ and order quantity for example $q \in (500, 650, 800, 850, 1000)$. The Monte Carlo simulation performance for the first 10 random attempts to level the ordered quantity q = 800. (Tab. 2)

Random trials	Ordered amount	Demand	Profit
	(q)	(<i>d</i>)	J
1.	800	999	80000
2.	800	746	7190
3.	800	842	8000
4	800	702	6530
5.	800	638	5570
6.	800	521	3815
7.	800	954	8000
8.	800	848	8000
9.	800	659	5885
10.	800	606	5090

Table 2 Simulation Monte Carlo for q=800

> d > q, if demand d = 999 and the quantity ordered q = 800, the value of profit is: Z = (30-20) q = 8000,

> d < q, if demand d = 746 and the quantity ordered q = 800, the value of profit is: Z = 30d - 20q + 15(q - d) = 7190.

The average value gain of ten random experiments is $6608 \in$ and the maximum profit under these conditions is $8000 \in$ and minimum profit is $3500 \in$.

The average profit calculated simulation of a hundred thousand repetitions of the quantity ordered $q \in (500,650,800,850,1000)$ in Tables 3 and 4. The maximum average profit is achieved when ordering quantity 800 packs of goods.

Tuble 5 Simulation results for 100 random trials						
Ordered	500	650	800	850	1000	
amount	500	050	000	0.50	1000	
Average profit (eur)	5000	6017	6646	6401	6516	

 Table 3 Simulation results for 100 random trials

Table 4 Simulation results for 100 random trials

Ordered amount	500	650	800	850	1000
Average profit (eur)	5000	6151	6626	6644	6144

The maximum profit is achieved when the amount of 800 is ordered. It is depicted in the graph.(Fig.4) and Tables 5 and 6.

	Probability			
Profit (eur)	10 random trials	100 random trials	1000 random trials	
3000 - 4000	0	0,04	0,069	
4000 - 5000	0,1	0,14	0,137	
5000 - 6000	0,1	0,12	0,134	
6000 - 7000	0,3	0,13	0,152	
7000 - 8000	0,5	0,57	0,508	

Table 6 Probability distribution for $a=100$	able	e 6 Pi	robabilitv	distribution	for	a = 100
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	Probability			
Drofit (our)	10	100	1000	
1 roju (eur)	random	random	random	
	trials	trials	trials	
3000 - 4000	0,1	0,11	0,137	
4000 - 5000	0,2	0,13	0,142	
5000 - 6000	0,3	0,13	0,137	
6000 - 7000	0,2	0,15	0,119	
7000 - 8000	0	0,15	0,137	
8000 - 9000	0,2	0,33	0,328	

The value of 95% confidence interval for the profit is:

- \blacktriangleright if q=500 : Z = 5000
- if q=600: Z ∈ (5783, 5967)
- ➤ if q=700: Z ∈ (7091, 7242)
- \succ if q=800: *Z* ∈ (6041, 6852)
- ➤ if q=900: Z ∈ (6323, 7308)
- ▶ if q=1000: $Z \in (5761, 6850)$



Figure 4 Profit of the quantity ordered

Thus, increasing the number of random experiments, the probability distribution of the frequency of the expected profit to the actual expected value. (Fig. 5)



Figure 5 Graph of probability distribution

This example is illustrative demonstration of the principle of Monte Carlo method, thus reselection of the probability distribution of input elements is derived probability distribution of output variables in the model.

Conclusion

The article is an example of using the Monte Carlo, demonstrated the possibility of its usage in order to show a universal method of solving math problems. A simulation is an important and has a stable place in the production process and also in business practice. It is not a tool to solve all the problems, but could be used to quickly optimize and improve. The simulation is not cheap method because requires expensive software, computer time and especially specialists who can properly use it. Nevertheless, the benefits of simulation are many times higher than the cost of the simulation project.



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